CBSE Test Paper 04

Chapter 10 Vector Algebra

1. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ then, the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ is
   a. $\frac{-3}{2}$
   b. $\frac{-5}{2}$
   c. $\frac{3}{2}$
   d. $\frac{5}{2}$
2. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\left(2\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - 3\vec{b}\right)$ externally in the ratio 1 : 2. Also, show that P is the mid point of the line segment RQ.
   a. $5\vec{a} + 5\vec{b}$
   b. $5\vec{a} + 3\vec{b}$
   c. $3\vec{a} + 3\vec{b}$
   d. $3\vec{a} + 5\vec{b}$
3. If l, m and n are the direction cosines of a line, Direction ratios of the line are the numbers which are
   a. Inversely Proportional to the direction cosine l of the line
   b. Proportional to the direction cosines of the line
   c. Proportional to the direction cosine l of the line
   d. Inversely Proportional to the direction cosines of the line
4. $\vec{a}\ and - \vec{a}$ are
   a. orthogonal vectors
   b. not equal and not collinear
   c. equal
   d. collinear
5. The value of the expression $\left|\overrightarrow{a} \times \overrightarrow{b}\right|^2$ + $\left(\overrightarrow{a} . \overrightarrow{b}\right)^2$ is ________.
6. The unit vector perpendicular to the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ forming a right-handed system is ________.
7. If $\vec{a}$ is non-zero vector, then $(\vec{a}.\hat{i})\hat{i}$ + $(\vec{a}.\hat{j})\hat{j}$ + $(\vec{a}.\hat{k})\hat{k}$ equals ________.

8. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$
   a. $\vec{a} = -\frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$
   b. $\vec{a} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$
   c. $\vec{a} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} - \frac{2}{\sqrt{6}}\hat{k}$
   d. $\vec{a} = \frac{1}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$
9. Find the unit vector in the direction of the sum of the vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $4\hat{i} - 3\hat{j} + 2\hat{k}$.
10. Compute the magnitude of $\vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}$.
11. Vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $\left|\vec{b}\right| = \frac{2}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector. Find angle between $\vec{a}$ and $\vec{b}$.
12. $\vec{a}$ Is unit vector and $(\vec{x} - \vec{a})(\vec{x} + \vec{a}) = 8$. Then find $|\vec{x}|$.
13. Find angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}| = 1, \left|\vec{b}\right| = 2, \vec{a}.\vec{b} = 1$.
14. The vectors $\vec{a} = 3\hat{i} + x\hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + y\hat{k}$ are mutually $\perp$. Given $|\vec{a}| = \left|\vec{b}\right|$ find x and y
15. Vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3, |\vec{b}| = 5$ and $|\vec{c}| = 7$. Find the angle between $\vec{a}$ and $\vec{b}$.
16. Show that the points $A\left(2\hat{i} - \hat{j} + \hat{k},\right), B\left(\hat{i} - 3\hat{j} - 5\hat{k}\right), C\left(3\hat{i} - 4\hat{j} - 4\hat{k}\right)$ are the vertices of a right-angled triangle.
17. If $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{b}$ in the form $\vec{b} = \vec{b}_1 + \vec{b}_2$, where $\vec{b}_1 \parallel \vec{a}$ and $\vec{b}_2 \perp \vec{a}$.
18. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{p}$, which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p} \cdot \vec{c} = 18$.

CBSE Test Paper 04

Chapter 10 Vector Algebra

Solution

1. a. $\frac{-3}{2}$

   Explanation: It is given that: If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, then: $(\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = \vec{0}.\vec{0}$
   $\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$
   $\Rightarrow 1 + 1 + 1 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$
   $\Rightarrow (\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = -\frac{3}{2}$

2. d. $3\vec{a} + 5\vec{b}$

   Explanation: Let position vector of point R be $\overrightarrow{r}$. As point R divides externally the line segment PQ in the ratio 1:2 .therefore ,
   $\overrightarrow{r} = \frac{1\left(\overrightarrow{a} - 3\overrightarrow{b}\right) - 2\left(2\overrightarrow{a} + \overrightarrow{b}\right)}{1-2} = \frac{\left(-3\overrightarrow{a} - 5\overrightarrow{b}\right)}{-1}$
   $\overrightarrow{r} = 3\overrightarrow{a} + 5\overrightarrow{b}$

   Also , mid point of the line segment RQ is :
   $= \frac{3\overrightarrow{a} + 5\overrightarrow{b} + \overrightarrow{a} - 3\overrightarrow{b}}{2} = \frac{4\overrightarrow{a} + 2\overrightarrow{b}}{2} = 2\overrightarrow{a} + \overrightarrow{b}$,which is the position vector of point P.Therefore , P is the mid point of line segment RQ.

3. b. Proportional to the direction cosines of the line

   Explanation: If l, m and n are the direction cosines of a line, Direction ratios of the line are the numbers which are Proportional to the direction cosines of the line.

4. d. collinear

   Explanation: $\vec{a}\ \ and - \vec{a}$are collinear vectors , because they are parallel in direction and having the same magnitude.

5. b. $\vec{a} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$

   Explanation: We have : $vector\overrightarrow{a} = \hat{i} + \hat{j} + 2\hat{k}$,

$$\widehat{a}=\frac{\overrightarrow{a}}{\left|\overrightarrow{a}\right|}=\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{1^2+1^2+2^2}}=\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}=\frac{\hat{i}}{\sqrt{6}}+\frac{\hat{j}}{\sqrt{6}}+\frac{2\hat{k}}{\sqrt{6}}$$

6. $\left|\overrightarrow{a}\right|^2\left|\overrightarrow{b}\right|^2$
7. $\hat{k}$
8. $\vec{a}$

9. Here,we put the given vectors equal to $\vec{a}=2\hat{i}+3\hat{j}-k$ and $\vec{b}=4\hat{i}-3\hat{j}+2\hat{k}$

   Now, sum of two vectors,

   $\vec{a}+\vec{b}=(2\hat{i}+3\hat{j}-\hat{k})+(4\hat{i}-3\hat{j}+2\hat{k})=6\hat{i}+\hat{k}$

   $\therefore$ Required unit vector = $\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|}$

   $=\frac{6\hat{i}+\hat{k}}{\sqrt{6^2+1^2}}=\frac{6\hat{i}+\hat{k}}{\sqrt{36+1}}=\frac{6\hat{i}+\hat{k}}{\sqrt{37}}=\frac{6}{\sqrt{37}}\hat{i}+\frac{\hat{k}}{\sqrt{37}}$

10. $\left|\vec{b}\right|=\sqrt{(2)^2+(-7)^2+(-3)^2}$

    $=\sqrt{4+49+9}$

    $=\sqrt{62}$

11. $sin\theta=\frac{|\vec{a}\times\vec{b}|}{|\vec{a}|\times|\vec{b}|}$

    $=\frac{1}{3\times\frac{2}{3}}=\frac{1}{2}$

    $\Rightarrow\theta=30^\circ$

12. $|\vec{a}|=1$

    $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=8$

    $|\vec{x}|^2-|\vec{a}|^2=8$

    $|\vec{x}|^2-1=8$

    $|\vec{x}|^2=9$

    $|\vec{x}|=3$

13. $\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}|\left|\vec{b}\right|}$

    $\cos\theta=\frac{1}{(1)(2)}=\frac{1}{2}$

    $\cos\theta=\cos\frac{\pi}{3}\Rightarrow\theta=\frac{\pi}{3}$

14. $since\ \vec{a}\perp\vec{b}$

$\vec{a}.\vec{b}=0$

6 - x - y = 0

y + x = 6 ....(1)

$|\vec{a}|=\left|\vec{b}\right|$ (given)

$3^2 + x^2 + 1 = 2^2 + 1^2 + y^2$

$y^2 - x^2 = 5$

(y - x)(y + x) = 5

6(y - x) = 5

$y-x=\frac{5}{6}$ .....(2)

From (1) and (2),we get,

$x=\frac{31}{12}, y=\frac{41}{12}$

15. According to the question ,

$|\vec{a}|=3,\ |\vec{b}|=5\text{ and }\ |\vec{c}|=7.$

Let the angle between $\vec{a}$ and $\vec{b}$ is θ.

Also, $\vec{a}+\vec{b}+\vec{c}=0\Rightarrow\vec{a}+\vec{b}=-\vec{c}$

$\Rightarrow\quad(\vec{a}+\vec{b})^2=(-\vec{c})^2$ [squaring both sides]

$\Rightarrow\quad(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=(-\vec{c})\cdot(-\vec{c})$

$\Rightarrow\quad\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}.\vec{b}=\vec{c}.\vec{c}$

$\Rightarrow|\vec{a}|^2+\vec{a}.\vec{b}+\vec{a}.\vec{b}+|\vec{b}|^2=|\vec{c}|^2$ $[\because\vec{x}.\vec{x}=|\vec{x}|^2$ and $\vec{a}.\vec{b}=\vec{b}\cdot\vec{a}]$

$\Rightarrow\quad|\vec{a}|^2+2\vec{a}.\vec{b}+|\vec{b}|^2=|\vec{c}|^2$

$\Rightarrow\quad|\vec{a}|^2+2|\vec{a}||\vec{b}|\cos\theta+|\vec{b}|^2=|\vec{c}|^2\quad\ldots(\text{i})\,[\because\vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta]$

$\Rightarrow\quad(3)^2+2\times3\times5\cos\theta+(5)^2=(7)^2$

$\Rightarrow\quad9+30\cos\theta+25=49$

$\Rightarrow\quad30\cos\theta=49-34$

$\Rightarrow\quad\cos\theta=\frac{15}{30}=\frac{1}{2}$

$\Rightarrow\quad\cos\theta=\cos\frac{\pi}{3}\left[\because\frac{1}{2}=\cos\frac{\pi}{3}\right]$

$\Rightarrow\quad\theta=\frac{\pi}{3}$

16. $\overrightarrow{AB}=-\hat{i}-2\hat{j}-6\hat{k}$

$\overrightarrow{BC}=2\hat{i}-\hat{j}+\hat{k}$

$\overrightarrow{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$

$|\overrightarrow{AB}|^2 = 1^2 + 2^2 + 6^2 = 41$

$|\overrightarrow{BC}|^2 = 2^2 + (-1)^2 + 1^2 = 6$

$|\overrightarrow{CA}|^2 = (-1)^2 + 3^2 + 5^2 = 35$

Here, $|\overrightarrow{AB}|^2 = |\overrightarrow{BC}|^2 + |\overrightarrow{CA}|^2$

Hence, the Δ is a right angled triangle.

17. According to the question,

$\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$

Let $\overrightarrow{b_1} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\overrightarrow{b_2} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$\overrightarrow{b_1} + \overrightarrow{b_2} = \vec{b}$, $\overrightarrow{b_1} \| \vec{a}$ and $\overrightarrow{b_2} \perp \vec{a}$.

Consider, $\overrightarrow{b_1} + \overrightarrow{b_2} = \vec{b}$

$\Rightarrow \quad (x_1 + x_2)\hat{i} + (y_1 + y_2)\hat{j} + (z_1 + z_2)\hat{k} = 2\hat{i} + \hat{j} - 3\hat{k}$

On comparing the coefficient of $\hat{i}, \hat{j}$ and $\hat{k}$ both sides; we get

$\Rightarrow x_1 + x_2 = 2$ ...(i)

$y_1 + y_2 = 1$...(ii)

and $z_1 + z_2 = -3$ ...(iii)

Now, consider $\overrightarrow{b_1} \| \vec{a}$

$\Rightarrow \quad \frac{x_1}{3} = \frac{y_1}{-1} = \frac{z_1}{0} = \lambda(\text{say})$

$\Rightarrow \quad x_1 = 3\lambda, y_1 = -\lambda$ and $z_1 = 0$ ...(iv)

On substituting the values of x,y and z, form Eq. (iv) to Eq. (i), (ii) and (iii), respectively, we get

$x_2 = 2 - 3\lambda, y_2 = 1 + \lambda$ and $z_2 = -3$ ...(v)

Since, $\overrightarrow{b_2} \perp \vec{a}$, therefore $\vec{b_2} \cdot \vec{a} = 0$

$\Rightarrow \quad 3x_2 - y_2 = 0$

$\Rightarrow \quad 3(2 - 3\lambda) - (1 + \lambda) = 0$

$\Rightarrow \quad 6 - 9\lambda - 1 - \lambda = 0$

$\Rightarrow \quad 5 - 10\lambda = 0 \Rightarrow \lambda = \frac{1}{2}$

On substituting $\lambda = \frac{1}{2}$ in Eqs. (iv) and Eqs. (iv) and (v), we get

$x_1 = \frac{3}{2}, y_1 = \frac{-1}{2}, z_1 = 0$

and $x_2 = \frac{1}{2}, y_2 = \frac{3}{2}$ and $z_2 = -3$

Hence, $\vec{b} = \vec{b}_1 + \vec{b}_2 = \left(\frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}\right) + \left(\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}\right)$

$= 2\hat{i} + \hat{j} - 3\hat{k}$

18. According to the question vectors are

$\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k},$

$\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$

$\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

Suppose, $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$

We have, $\vec{p}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.

$\vec{p} \cdot \vec{a} = 0$

$\Rightarrow \quad (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + 4\hat{j} + 2\hat{k}) = 0$

$\Rightarrow \quad x + 4y + 2z = 0$ ...(i)

and $\vec{p}.\vec{b} = 0$

$\Rightarrow \quad (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (3\hat{i} - 2\hat{j} + 7\hat{k}) = 0$

$\Rightarrow \quad 3x - 2y + 7z = 0$ ...(ii)

Also, given $\vec{p} \cdot \vec{c} = 18$

$\Rightarrow \quad \left(x\hat{i} + y\hat{j} + z\hat{k}\right) \cdot (2\hat{i} - \hat{j} + 4\hat{k}) = 0$

$\Rightarrow \quad 2x - y + 4z = 18$ ...(iii)

Multiplying Eq. (i) by 3 and subtracting it from Eq. (ii), we get

$-14y + z = 0$

Multiplying Eq. (i) by 2 and subtracting it from Eq. (iii), we get

$-9y = 18$

$\Rightarrow y = -2$

On putting y = -2 and z= -28 in Eq. (i), we get

$x + 4(-2) + 2(-28) = 0$

$\Rightarrow x - 8 - 56 = 0$

$\Rightarrow x = 64$

Hence, the required vector is

$\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$

i.e. $\vec{p} = 64\hat{i} - 2\hat{j} - 28\hat{k}$