CBSE Test Paper 05

Chapter 4 Determinants

1. Solution set of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ is
   a. None of these
   b. {2, 5, 6}
   c. {1, 2, 7}
   d. {–9, 2, 7}
2. If A is a non singular matrix of order 3 , then $|\text{adj}(A^3)|$ =.
   a. None of these
   b. $|A|^8$
   c. $|A|^6$
   d. $|A|^9$
3. Solution set of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$ is
   a. {2, 1, 5}
   b. {2, 0,1}
   c. {–3, 1, 5}
   d. {2, –3,1}
4. For an invertible square matrix of order 3 with real entries $A^{-1} = A^2$, then det. A =.
   a. $\frac{1}{3}$
   b. 3
   c. None of these
   d. 1
5. If A and B are square matrices of order 3, such that Det.A = –1, Det.B =3 then, the determinant of 3AB is equal to
   a. -27
   b. -81
   c. -9

d. 81

6. The sum of the products of elements of any row with the co-factors of corresponding elements is equal to ________.
7. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then value of x is ________.
8. If the area of a triangle with vertices (-3, 0), (3, 0) and (0, k) is 9 sq. units, the value of k will be ________.
9. If A = $= \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ then write A^{-1} in terms of A.
10. Write A^{-1} for A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.
11. What positive value of x makes following pair of determinants equal?
$\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}, \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$
12. Without expanding, prove that $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0.$
13. If B = [-7], find det B.
14. Find value of k if area of triangle is 4 sq. units and vertices are :(k ,0), (4, 0), (0,2).
15. $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, Find the no. a and b such that $A^2 + aA + bI = 0$ Hence find A^{-1}.
16. Prove that $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k).$
17. Prove that $\Delta = \begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}.$
18. Given $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$.Find AB and use this result in solving the following system of equations.
$x - y + z = 4$
$x - 2y - 2z = 9$
$2x + y + 3z = 1$

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Solution

1. d. { –9 ,2 , 7 }, **Explanation:** x(x^2 - 12) - 3(2x -14) + 7(12 - 7x) = 0

   $\Rightarrow$ x^3 - 67x +126 =0

   $\Rightarrow$ (x - 2)(x - 7)(x + 9) = 0 $\Rightarrow$ x = 2, 7, -9

2. c. $|A|^6$, **Explanation:** If A is a non singular matrix of order 3, then $|adj(A^3)| = (|A^3|)^2$

   $= (|AAA|)^2 = (|A|\ |A|\ |A|)^2 = (|A|^3)^2 = |A|^6$.

3. d. {2, -3, 1}, **Explanation:** Expanding along R_1

   [x(-3x(x+2) - 2x(x-3)] + 6[2(x +2) + 3(x - 3)] -1 (4x - 9x) = 0

   $\Rightarrow$ $-5x^3$ + 35 x - 30 = 0 $\Rightarrow$ (x - 1)(x - 2)(x + 3) = 0 $\Rightarrow$ x = 1, 2, -3

4. d. 1, **Explanation:** $A^2 = I \Rightarrow A^2A^{-1} = IA^{-1} \Rightarrow A = A^{-1}$ and it is possible only if A is an identity matrix and determinant of identity matrix is equal to 0

5. b. -81, **Explanation:** $|3AB| = 3^3|A||B| = 27(-1)(3) = -81$

6. value of determinant
7. ± 6
8. 3
9. We have, $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

   Clearly, adj A = $\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$

   and $|A| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -4 - 15 = -19$

   $\therefore \quad A^{-1} = \frac{1}{|A|}$ adj $A = \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$

   $\equiv \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \frac{1}{19}A$

10. We have, A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

Clearly, adj A = $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$ and $|A| = \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = 6 - 5 = 1$

$\therefore \quad A^{-1} = \frac{1}{|A|}$ adj $(A) = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

11. Let $\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$

On expanding, we get

$2x^2$ - 15= 32 - 15

$\Rightarrow 2x^2$ - 15 = 17

$\Rightarrow 2x^2$ - 32 $\Rightarrow x^2$ = 16

$\Rightarrow$ x = ± 4

Hence, for x = 4, given pair of determinants is equal.

12. Applying $R_1 \to R_1 + R_2$,we get,

$$\Delta = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

Taking x+y+z common from R_1

$\Delta = (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ = 0 $[\because R_1 \ and \ R_3 \ are \ identical]$

13. |B| = 7 [since |a| = a, for some constant a]

14. Given: Area of triangle =$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 4$ sq. units

$\Rightarrow \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = \pm 4$

$\Rightarrow \frac{1}{2}[k(0-2) - 0 + 1(8-0)] = \pm 4$

$\Rightarrow \frac{1}{2}(-2k+8) = \pm 4$

$\Rightarrow -k + 4 = \pm 4$

$\Rightarrow -k + 4 = \pm 4$

Taking positive sign, $-k + 4 = 4 \Rightarrow k = 0$

Taking negative sign, $-k + 4 = -4 \Rightarrow k = 8$

15. Here,$A^2 = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$

$$A^2 + aA + bI = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 11+3a+b & 8+2a \\ 4+a & 3+a+b \end{bmatrix}$$

By the question, $\begin{bmatrix} 11+3a+b & 8+2a \\ 4+a & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

a = -4, b =1

$A^2 - 4A + I = 0$

$A^2 - 4A = -I$

$AAA^{-1} - 4AA^{-1} = -IA^{-1}$

$A - 4I = -A^{-1}$

$A^{-1} = 4I - A$

$$= \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

16. $L.H.S = \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} \quad [C_1 \to C_1 + C_2 + C_3]$

$$= \begin{vmatrix} 3y+k & y & y \\ 3y+k & y+k & y \\ 3y+k & y & y+k \end{vmatrix}$$

Taking 3y + k common from C_1

$$= (3y+k)\begin{vmatrix} 1 & y & y \\ 1 & y+k & y \\ 1 & y & y+k \end{vmatrix}$$

[operating $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$]

$$= (3y+k)\begin{vmatrix} 1 & y & y \\ 0 & k & 0 \\ 0 & 0 & k \end{vmatrix}$$

$$= (3y+k).1\begin{vmatrix} k & 0 \\ 0 & k \end{vmatrix}$$

$= (3y + k).k^2 = k^2(3y + k)$

= R.H.S. Proved.

17. Applying $R_1 \to R_1 - xR_3$,we get,

$$\Delta = \begin{vmatrix} a\left(1-x^2\right) & c\left(1-x^2\right) & p\left(1-x^2\right) \\ ax+b & cx+d & bx+q \\ u & v & w \end{vmatrix}$$

Taking $(1-x^2)$ common from R_1 ,we get,

$$\Delta = (1-x^2)\begin{vmatrix} a & c & p \\ ax+b & cx+d & bx+q \\ u & v & w \end{vmatrix}$$

$$Applying\ R_2 \to R_2 - xR_1, we\ get,$$

$$\Delta = (1-x^2)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$$

18. Given System of Equations,

$$x-y+z=4,\ x-2y-2z=9, 2x+y+3z=1$$

Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$

Then given system of equations can be rewritten as, AX = C

$$AB = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

Now, AB = 8I

$$A^{-1} = \frac{1}{8}B \begin{bmatrix} \because A^{-1}AB = 8A^{-1}I \\ B = 8A^{-1} \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{-7}{8} & \frac{1}{8} & \frac{3}{8} \\ \frac{5}{8} & \frac{-3}{8} & \frac{-1}{8} \end{bmatrix}$$

Now, AX = C

$\Rightarrow$ X = A^{-1}C $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{-7}{8} & \frac{1}{8} & \frac{3}{8} \\ \frac{5}{8} & \frac{-3}{8} & \frac{-1}{8} \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$

$\Rightarrow$ x = 3, y = - 2, z = - 1