

CBSE Test Paper 01
Chapter 1 Relations and Functions

1. In Z , the set of integers, inverse of -7 , w.r.t. ' $*$ ' defined by $a * b = a + b + 7$ for all $a, b \in Z$, is
 - a. -7
 - b. -14
 - c. 14
 - d. 7
2. If $A = \{1, 2, 3\}$, then the relation $R = \{(1, 2), (2, 3), (1, 3)\}$ in A is ____.
 - a. transitive only
 - b. reflexive only
 - c. symmetric only
 - d. symmetric and transitive only
3. A relation R on a set A is called an empty relation if
 - a. no element of A is related to any element of A
 - b. every element of A is related to one element of A
 - c. one element of A is related to all the elements of A
 - d. every element of A is related to any element of A
4. Let f and g be two functions from R to R defined as $f(x) = \begin{cases} 0, & x \text{ is rational} \\ 1, & x \text{ is irrational} \end{cases}$,
 $g(x) = \begin{cases} -1, & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$ then, $(g \circ f)(e) + (f \circ g)(\pi) =$.
 - a. 1
 - b. 2
 - c. -1
 - d. 0
5. Let R be the relation on N defined as xRy if $x + 2y = 8$. The domain of R is
 - a. $\{2, 4, 6, 8\}$
 - b. $\{2, 4, 8\}$
 - c. $\{1, 2, 3, 4\}$
 - d. $\{2, 4, 6\}$
6. If $n(A) = p$ and $n(B) = q$, then the number of relations from set A to set $B =$ _____.

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7. A function is called an onto function, if its range is equal to _____.
8. A binary operation $*$ on a set X is said to be _____, if $a * b = b * a$, where $a, b \in X$.
9. Find gof $f(x) = |x|$, $g(x) = |5x + 1|$.
10. Show that function $f: \mathbb{N} \rightarrow \mathbb{N}$, given by $f(x) = 2x$, is one – one.
11. Let $S = \{1, 2, 3\}$ Determine whether the function $f: S \rightarrow S$ defined as below have inverse.
 $f = \{(1, 1), (2, 2), (3, 3)\}$
12. Let $f: \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g: \{1, 2, 5\} \rightarrow \{1, 3\}$ be given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down gof .
13. Consider $f: \{1, 2, 3\} \rightarrow \{a, b, c\}$ given by $f(1) = a$, $f(2) = b$ and $f(3) = c$ find f^{-1} and show that $(f^{-1})^{-1} = f$.
14. Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function? If g is described by $g(x) = ax + b$, then what value should be assigned to a and b .
15. Show that the relation R defined by $(a, b) R (c, d) \Rightarrow a + d = b + c$ on the set $\mathbb{N} \times \mathbb{N}$ is an equivalence relation.
16. Let the function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \cos x$, $\forall x \in \mathbb{R}$. Show that f is neither one-one nor onto.
17. Let L be the set of all lines in plane and R be the relation in L define if $R = \{(l_1, l_2) : l_1 \text{ is } \perp \text{ to } l_2\}$. Show that R is symmetric but neither reflexive nor transitive.
18. If the function $f: \mathbb{R} \rightarrow \mathbb{R}$ is given by $f(x) = x^2 + 2$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ is given by $g(x) = \frac{x}{x-1}$; $x \neq 1$ then find fog and gof and hence find $\text{fog}(2)$ and $\text{gof}(-3)$.

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Solution

1. a. -7

Explanation: If 'e' is the identity, then $a * e = a \Rightarrow a + e + 7 = a \Rightarrow e = -7$.

Also, inverse of e is e itself. Hence, inverse of -7 is -7.

2. a. transitive only

Explanation: A relation R on a non-empty set A is said to be transitive if xRy and $yRz \Rightarrow xRz$, for all $x \in R$. Here, (1, 2) and (2, 3) belongs to R implies that (1, 3) belongs to R.

3. a. no element of A is related to any element of A

Explanation: For any set A, an empty relation may be defined on A as: there is no element exists in the relation set which satisfies the relation for a given set A i.e.

let $A = \{1, 2, 3, 4, 5\}$ and $R = \{(a, b) : a, b \in A \text{ and } a + b = 10\}$, so we get $R = \{ \}$ which is an empty relation.

4. c. -1

Explanation: $(g \circ f)(e) + (f \circ g)(\pi) = g(f(e)) + f(g(\pi)) = g(1) + f(0) = -1 + 0 = -1$.

5. d. {2, 4, 6}

Explanation: As xRy if $x + 2y = 8$, therefore, domain of the relation R is given by $x = 8 - 2y \in \mathbb{N}$. When $y = 1, \Rightarrow x = 6$, when $y = 2, \Rightarrow x = 4$, when $y = 3, \Rightarrow x = 2$. Therefore, domain is { 2, 4, 6 }.

6. 2^{pq}

7. codomain

8. commutative

9. $g \circ f(x) = g[f(x)]$

$$= g[|x|]$$

$$= |5|x| + 1|$$

10. For, $f(x_1) = f(x_2)$

$$2x_1 = 2x_2$$

$$x_1 = x_2$$

So, The function f is one – one

11. Since different elements have different images. So, f is one - one. Also, every element of codomain has pre-image so, f is onto

Now f is one – one and onto, so that f is invertible with inverse $f^{-1} = \{(1, 1) (2, 2) (3, 3)\}$

12. $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$

Now, $f(1) = 2, f(3) = 5, f(4) = 1$ and $g(1) = 3, g(2) = 3, g(5) = 1$

$$g \circ f(x) = g(f(x))$$

$$g \circ f(1) = g(f(1)) = g(2) = 3$$

$$g[f(3)] = g(5) = 1 \text{ and } g[f(4)] = g(1) = 3$$

Hence, $g \circ f = \{(1, 3), (3, 1), (4, 3)\}$

13. $f = \{(1, a) (2, b) (3, c)\}$

$$f^{-1} = \{(a, 1) (b, 2) (c, 3)\}$$

$$(f^{-1})^{-1} = \{(1, a) (2, b) (3, c)\}$$

Hence $(f^{-1})^{-1} = f$.

14. Yes, g is a function since every element in domain has a unique image in range.

Now, Let $g(x) = ax + b$ Then Given,

$$g(1) = a + b = 1 \text{ \&}$$

$$g(2) = 2a + b = 3$$

Subtracting $g(1)$ from $g(2)$ Gives

$$(2a + b) - (a + b) = a = 2 \text{ \& Substituting It into } g(1)$$

We have $b = -1$

15. we have, $a + b = b + a$ for all $(a, b) \in N \times N$, which implies $(a, b) R (a, b)$. Thus, R is reflexive.

Let $(a, b), (c, d) \in N \times N$ be such that

$$(a, b) R (c, d)$$

$$\Rightarrow a + d = b + c$$

$$\Rightarrow d + a = c + b$$

$$\Rightarrow c + b = d + a$$

$\Rightarrow (c, d) R (a, b)$ for all $(a, b), (c, d) \in N \times N$

Hence R is symmetric.

Let $(a,b),(c,d),(e,f) \in N \times N$ such that $(a,b) R (c,d)$ and $(c,d) R (e,f)$. Then,

$$(a, b) R (c, d) \Rightarrow a + d = b + c \dots\dots (1)$$

$$(c, d) R (e, f) \Rightarrow c + f = d + e \dots\dots(2)$$

Adding (1) and (2)

$$(a + d) + (c+f) = (b + c) + (d + e)$$

$$a + f = b + e$$

$$(a, b) R (e, f)$$

Hence, R is transitive

So, R is an equivalence relation.

16. Given function, $f(x) = \cos x, \forall x \in R$

$$\text{Now, } f\left(\frac{\pi}{2}\right) = \cos \frac{\pi}{2} = 0$$

$$\Rightarrow f\left(\frac{-\pi}{2}\right) = \cos \frac{\pi}{2} = 0$$

$$\Rightarrow f\left(\frac{\pi}{2}\right) = f\left(\frac{-\pi}{2}\right)$$

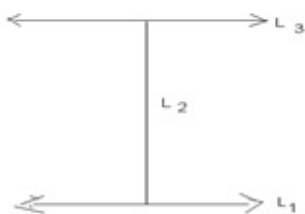
$$\text{But } \frac{\pi}{2} \neq \frac{-\pi}{2} = 0$$

So, $f(x)$ is not one-one

Now, $f(x) = \cos x, \forall x \in R$ is not onto as there is no pre-image for any real number.

Which does not belong to the intervals $[-1, 1]$, the range of $\cos x$.

17. R is not reflexive, as a line L_1 cannot be \perp to itself i.e $(L_1, L_1) \notin R$



Now $(L_1, L_2) \in R$

$$\Rightarrow L_1 \perp L_2$$

$$\Rightarrow L_2 \perp L_1$$

$$\Rightarrow (L_2, L_1) \in R$$

R is symmetric

Now (L_1, L_2) and $(L_2, L_3) \in R$

i.e $L_1 \perp L_2$ and $L_2 \perp L_3$

Then L_1 can never be \perp to L_3 in fact $L_1 \parallel L_3$

i.e $(L_1, L_2) \in R, (L_2, L_3) \in R.$

But $(L_1, L_3) \notin R$

R is not transitive.

18. We are given that, $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2 + 2$ and $g(x) = \frac{x}{x-1}; x \neq 1.$

First we see whether $f \circ g$ and $g \circ f$ exist for the given functions.

Since, $\text{range } f \subseteq \text{domain } g$ and $\text{range } g \subseteq \text{domain } f$

Hence, $f \circ g$ and $g \circ f$ exist for the given functions.

Now, for any $x \in \mathbb{R} - \{1\}$, we have $(f \circ g)(x) = f[g(x)]$

$$\begin{aligned} &= f\left[\frac{x}{x-1}\right] = \left(\frac{x}{x-1}\right)^2 + 2 \\ &= \frac{x^2 + 2(x-1)^2}{(x-1)^2} = \frac{x^2 + 2(x^2 + 1 - 2x)}{(x-1)^2} \\ &= \frac{3x^2 + 2 - 4x}{(x-1)^2} \end{aligned}$$

$\therefore f \circ g : \mathbb{R} \rightarrow \mathbb{R}$ is defined by

$$(f \circ g)(x) = \frac{3x^2 - 4x + 2}{(x-1)^2}, x \neq 1 \dots\dots(i)$$

For any $x \in \mathbb{R}$ we have

$$\begin{aligned} g \circ f(x) &= g[f(x)] \\ &= g(x^2 + 2) = \frac{x^2 + 2}{(x^2 + 2) - 1} = \frac{x^2 + 2}{x^2 + 1} \end{aligned}$$

$\therefore g \circ f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by

$$(g \circ f)(x) = \frac{x^2 + 2}{x^2 + 1} \dots\dots(ii)$$

On putting $x = 2$ in Eq. (i), we get

$$\begin{aligned} f \circ g(2) &= \frac{3 \times (2)^2 - 4(2) + 2}{(2-1)^2} = \frac{3 \times 4 - 8 + 2}{(1)^2} \\ &= 12 - 8 + 2 = 6 \end{aligned}$$

On putting $x = -3$ in Eq. (ii), we get

$$g \circ f(-3) = \frac{(-3)^2 + 2}{(-3)^2 + 1} = \frac{9 + 2}{9 + 1} = \frac{11}{10}$$