CBSE Test Paper 05

Chapter 1 Real Numbers

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1. The exponent of 3 in the prime factorization of 864 is: **(1)**
    a. 2
    b. 3
    c. 4
    d. 8
2. Which of the following numbers has terminating decimal expansion? **(1)**
    a. $\frac{3}{11}$
    b. $\frac{3}{7}$
    c. $\frac{3}{5}$
    d. $\frac{5}{3}$
3. The decimal expansion of $\frac{987}{10500}$ will terminate after: **(1)**
    a. 2 decimal places
    b. 3 decimal places
    c. 1 decimal place
    d. None of these
4. The smallest number of 4 digits exactly divisible by 12, 15, 18 and 27 is **(1)**
    a. 1000
    b. 1080
    c. 1002
    d. 1001
5. A number when divided by 61 gives 27 as quotient and 32 as remainder, then the number is: **(1)**
    a. 1796
    b. 1569
    c. 1679
    d. 1967
6. Given that HCF (306, 657) = 9, find LCM (306, 657). **(1)**
7. State whether $\frac{129}{2^2 \times 5^7 \times 7^{17}}$ will have terminating decimal expansion or a non-terminating repeating decimal expansion. **(1)**

8. Express the given number as product of its prime factors: 234. **(1)**
9. Factorise and find the HCF of the following pairs of polynomials: **(1)**
$4x^3\left(x^3+27\right)$ and $10x^5\left(x^2+6x+9\right)$
10. What can you say about the prime factorisations of the denominators of $27.\overline{142857}$ rational. **(1)**
11. Prove that $\sqrt{3}+\sqrt{5}$ is irrational. **(2)**
12. Without actual division, show that $\frac{129}{\left(2^2\times5^3\times7^2\right)}$ is a non terminating repeating decimal. **(2)**
13. Find the largest number which exactly divides 280 and 1245 leaving remainders 4 and 3, respectively. **(2)**
14. Find the values of a and b so that the polynomials P(x) and Q(x) have $\left(x^2-x-12\right)$ as their HCF, where **(3)**
$P(x)=\left(x^2-5x+4\right)\left(x^2+5x+a\right)$
$Q(x)=\left(x^2+5x+6\right)\left(x^2-5x-2b\right)$
15. Prove that 3 + $\sqrt{5}$ is an irrational number. **(3)**
16. If $\frac{241}{4000}=\frac{241}{2^m5^n}$ find the values of m and n where m and n are non-negative integers. Hence, write its decimal expansion without actual division. **(3)**
17. A sweet seller has 420 kaju barfis and 130 badam barfis. She wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. What is the maximum number of barfis that can be placed in each stack for this purpose? **(3)**
18. Use Euclid's Division Lemma to show that the cube of any positive integer is of the form 9m, 9m + 1, or 9m + 8 for some integer m. **(4)**
19. Prove that the square of any positive integer is of the form 3m or, 3m + 1 but not of the form 3m + 2. **(4)**
20. Prove that if both x and y are positive odd integers, x = 2m + 1 and y = 2n + 1, prove that $x^2 + y^2$ is an even integer but not divisible by 4 where m and n are positive integer. **(4)**

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Solution

1. b. 3

   Explanation: Prime factorization of 864 = 32×27 = $2^5 \times 3^3$

   Therefore the exponent of 3 in the prime factorization of 864 is 3

2. c. $\frac{3}{5}$

   Explanation: $\frac{3}{5}$ has terminal decimal expansion because terminal decimal expansion should have the denominator 2 or 5 only.

3. b. 3 decimal places

   Explanation: $\frac{987}{10500}$ = $\frac{47}{500}$ = $\frac{47}{2^2 \times 5^3}$

   Here, in the denominator of the given fraction the highest power of prime factor 5 is 3, therefore, the decimal expansion of the rational number $\frac{47}{2^2 \times 5^3}$ will terminate after 3 decimal places.

4. b. 1080

   Explanation: LCM (12, 15, 18, 27) = 540

   Now, smallest four digit number = 1000

   $\therefore$ 1000 $\div$ 540 = 1 $\times$ 540 + 460 (Remainder = 460)

   Therefore, the smallest number of 4 digits exactly divisible by 12, 15, 18 and 27 is 1000 + (540 – 460) = 1000 + 80 = 1080

5. c. 1679

   Explanation: Dividend = Divisor$\times$ Quotient + Remainder

   $\rightarrow$ Number(dividend) = D $\times$ Q + R

   Therefore the number (Dividend) = 61$\times$27 + 32

   = 1647 + 32 = 1679

6. As we know that, HCF $\times$ LCM = Product of two numbers

   LCM (306, 657) = $\frac{306 \times 657}{HCF(306,657)}$ = $\frac{306 \times 657}{9}$ = 22338.

7. According to the question,

   The given number is $\frac{129}{2^2 \times 5^7 \times 7^{17}}$

Clearly, none of $2, 5\ and\ 7$ is a factor of 129.

So, the number will have a non terminating decimal expansion.

8. Using prime factorization, we have
   $234 = 2 \times 3 \times 3 \times 13 = 2 \times 3^2 \times 13$

9. $P(x) = 4x^3\left(x^3 + 27\right) = 4x^3\left(x^3 + 3^3\right)$
   $= 4\mathrm{x}^3(\mathrm{x}+3)\left(\mathrm{x}^2 - 3\mathrm{x} + 9\right)$
   Using Identity $\mathrm{a}^3 + \mathrm{b}^3 = (\mathrm{a}+\mathrm{b})\left(\mathrm{a}^2 - \mathrm{ab} + \mathrm{b}^2\right)$
   $\mathrm{Q(x)} = 10\mathrm{x}^5\left(\mathrm{x}^2 + 6\mathrm{x} + 9\right) = 10\mathrm{x}^5(\mathrm{x}+3)^2$
   Using Identity $(\mathrm{a}+\mathrm{b})^2 = \mathrm{a}^2 + 2\mathrm{ab} + \mathrm{b}^2$
   $\therefore$ HCF of $P(x)$ and $Q(x) = 2x^3(x+3)$

10. Let x = $\overline{27.142857}$ ......(i)

    1000000x = 27142857.142857 ......(ii)

    Subtract (i) from (ii)

    999999x = 27142830

    x = 27142830/999999

    999999 = 3 × 3 × 111111

11. Let us consider $\sqrt{3} + \sqrt{5}$ is a rational number that can be written as
    $\sqrt{3} + \sqrt{5}$ = a
    $\Rightarrow \sqrt{5} = a - \sqrt{3}$
    Squaring both sides, we get
    $(\sqrt{5})^2 = (a - \sqrt{3})^2$
    ⇒ 5 = (a)^2 $+(\sqrt{3})^2 - 2(a)(\sqrt{3})$
    ⇒ $2a\sqrt{3}$ = a^2 + 3 - 5
    ⇒ $2a\sqrt{3}$ = a^2 - 2
    $\Rightarrow \sqrt{3} = \frac{a^2-2}{2a}$
    As a^2 – 2, 2a are integers .
    So $\frac{a^2-2}{2a}$ is also rational but $\sqrt{3}$ is not rational which contradicts our consideration.
    Since a rational number cannot be equal to an irrational number . Our assumption that √3 + √5 is rational wrong .
    So, $\sqrt{3} + \sqrt{5}$ is irrational.

12. The given number is $\frac{129}{\left(2^2 \times 5^3 \times 7^2\right)}$.

Clearly, none of 2, 5 and 7 is a factor of 129.

So, the given rational is in its simplest form.

And, $\left(2^2 \times 5^3 \times 7^2\right) \neq \left(2^m \times 5^n\right)$

$\therefore \frac{129}{\left(2^2 \times 5^3 \times 7^2\right)}$ is a non-terminating repeating decimal.

As the given number contains a factor 7^2 in it's denominator so it is non-terminating repeating decimal.

13. We need to find the largest number which exactly divides 280 and 1245 leaving remainders 4 and 3, respectively. The required number when divides 280 and 1245, leaves remainder 4 and 3, this means

280 - 4 = 276 and 1245 - 3 = 1242 are completely divisible by the number.

Therefore, the required number = H.C.F. of 276 and 1242.

By applying Euclid's division lemma:

1242 = 276 $\times$ 4 + 138

276 = 138 $\times$ 2 + 0.

Therefore, H.C.F of 276 and 1242 = 138.

Hence, the required number is 138.

14. $\mathrm{HCF} = \left(\mathrm{x}^2 - \mathrm{x} - 12\right) = (\mathrm{x}+3)(\mathrm{x}-4)$

$P(x) = \left(x^2 - 5x + 4\right)\left(x^2 + 5x + a\right)$

$= (x-4)(x-1)\left(x^2 + 5x + a\right)$

Since, $(x+3)(x-4)$ is the HCF of $P(x)$ and $Q(x)$therefore,

(x+3) and (x-4) are factors of p(x), As (x-4) is already seen

in p(x) and (x+3) is also a factor of p(x).

Thus, by factor theorem, x + 3 = 0 $\Rightarrow x = -3 i, e \cdot P(-3) = 0$

Hence, $P(-3) = (-7)(-4)(9 - 15 + a) = 0$

$\Rightarrow 28(-6 + a) = 0 \quad \Rightarrow a = 6$

Again, $\mathrm{Q}(\mathrm{x}) = \left(\mathrm{x}^2 + 5\mathrm{x} + 6\right)\left(\mathrm{x}^2 - 5\mathrm{x} - 2\mathrm{b}\right)$

$= (x+2)(x+3)\left(x^2 - 5x - 2b\right)$

Since, $x - 4$ is a factor of $Q(x)$

$x - 4 = 0 \Rightarrow x = 4$, by factor theorem Q(4) must equal to 0.

$Q(4) = (6)(7)(16 - 20 - 2b) = 0$

$\Rightarrow 42(-4-2b)=0 \Rightarrow 2b=-4 \Rightarrow b=-2$

Hence, a = 6, b = -2

15. Let 3 + $\sqrt{5}$ is a rational number.

$3+\sqrt{5}=\frac{p}{q}, q \neq 0$

$3+\sqrt{5}=\frac{p}{q}$

$\Rightarrow \quad \sqrt{5}=\frac{p}{q}-3$

$\Rightarrow \quad \sqrt{5}=\frac{p-3q}{q}$

Now in RHS $\frac{p-3q}{p}$ is rational

This shows that $\sqrt{5}$ is rational

But this contradict the fact that $\sqrt{5}$ is irrational, This is because we assumed that 3 + $\sqrt{5}$ is a rational number.

$\therefore 3+\sqrt{5}$ is an irrational number.

16. According to question,

$\frac{241}{4000}=\frac{241}{2^m 5^n}$

$\Rightarrow \frac{241}{2^5 \times 5^3}=\frac{241}{2^m 5^n}$

$\Rightarrow m=5, n=3$

Now $\frac{241}{4000}=\frac{241}{2^5 \times 5^3}$

$=\frac{241 \times 5^2}{2^5 \times 5^3 \times 5^2}$ (by multiplying and dividing by 5^2)

$=\frac{6025}{(2 \times 5)^5}$

$=\frac{6025}{(10)^5}$?

= 0.06025

17. A sweet seller has 420 kaju barfis and 130 badam barfis

HCF(420, 130) will give the maximum number of barfis that can be placed in each stack.

By Euclid's division algorithm,

$420=130 \times 3+30$

$130=30 \times 4+10$

$30=10 \times 3+0$

$So\ HCF(420,130)=10$

∴ The sweet seller can make stacks of 10 for both kinds of barfis.

18. we have to Use Euclid's Division Lemma to show that the cube of any positive integer is of the form 9m, 9m + 1, or 9m + 8 for some integer m.

Let $a = 3q + r, 0 \leq r < 3$

or a = 3q, 3q + 1 and 3q + 2

Case I : $a = 3q$

or $a^3 = (3q)^3 = 27q^3 = 9(3q^3)$

$= 9m$ where $m = 3q^3$

Case II : $a = 3q + 1$

$\Rightarrow a^3 = (3q + 1)^3$

$= 27q^3 + 9q(3q + 1) + 1$

$= 9m + 1$

where $m = 3q^3 + 3q^2 + 1$

Case III : $a = 3q + 2$

$\Rightarrow a^3 = (3q + 2)^3$

$= 27q^3 + 18q(3q + 2) + 8$

$= 9(3q^3 + 6q^2 + 4q) + 8$

$= 9m + 8$

where $m = 3q^2 + 6q^2 + 4q$

From Case I, II and III, we conclude that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8 for some integer m.

19. By Euclid’s division algorithm

$a = bq + r$, where $0 \leq r \leq b$

Put $b = 3$

$a = 3q + r$, where $0 \leq r \leq 3$

If $r = 0$, then $a = 3q$

If $r = 1$, then $a = 3q + 1$

If $r = 2$, then $a = 3q + 2$

Now, $(3q)^2 = 9q^2$

$= 3 \times 3q^2$

$= 3m$, where m is some integer

$(3q + 1)^2 = (3q)^2 + 2(3q)(1) + (1)^2$

$= 9q^2 + 6q + 1$

$= 3(3q^2 + 2q) + 1$

= 3m + 1, where m is some integer

$(3q + 2)^2 = (3q)^2 + 2(3q)(2) + (2)^2$

$= 9q^2 + 12q + 4$

$= 9q^2 + 12q + 4$

$= 3(3q^2 + 4q + 1) + 1$

= 3m + 1, where m is some integer

Hence the square of any positive integer is of the form 3m, or 3m +1

But not of the form 3m + 2

20. Since both x and y are positive odd integers x = 2m + 1 and y = 2n + 1, where m and n are some whole numbers.

$x^2 + y^2 = (2m + 1)^2 + (2n + 1)^2$

$= 4m^2 + 4m + 1 + 4n^2 + 4n + 1$

$= 4(m^2 + n^2 + m + n) + 2$

= 4q + 2, where $q = m^2 + m + n^2 + n$, which is a whole number.

We note that 4q + 2 is an even integer but leaves reminder 2 when divided by 4.

Hence, $x^2 + y^2$ is an even integer but not divisible by 4.